What can black holes tell us about microstates?

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Introduction and motivation

In the holographic description of gravity, the fundamental degrees of freedom are those of a quantum theory without gravity and gravity is an emergent phenomenon.

Nevertheless it will be useful to quantize gravity on AdS by standard tools, *e.g.* string theory, and explore how the holographic results emerge from this.

– could teach us how to quantize gravity in situations where holography is not directly applicable.

We shall study a version of this problem with extremal black holes whose near horizon geometry contains an AdS_2 factor.

By now there is considerable evidence that in string theory Bekenstein-Hawking entropy has a statistical interpretation.

1. Identify a suitable supersymmetric black hole with certain quantum numbers and calculate its entropy S_{BH} via the Bekenstein-Hawking-Wald formula.

2. Find a microscopic system with the same quantum numbers and count its 'number of states' Ω in the limit when gravity is switched off.

3. Compare S_{BH} with $\ln \Omega$.

This comparison is usually done in the limit when the charges are large.

$\Rightarrow$ the curvature at the horizon is small and hence the Bekenstein-Hawking formula is a reliable approximation.

Also the counting of states simplifies in this limit since we can use asymptotic formula, *e.g.* the Cardy formula.

What happens beyond the large charge limit?

On the microscopic side the low energy dynamics is usually described by a supersymmetric quantum mechanics and there is no difficulty in principle in counting states to arbitrary accuracy.

In a class of N=4 and N=8 supersymmetric string theories one now has exact microscopic results.

Dijkgraaf, Verlinde, Verlinde; Shih, Strominger, Yin; David, Jatkar, A.S.

Can we understand these results by directly studying quantum gravity / string theory corrections to the black hole entropy?

Some results for the microscopic index in heterotic on T^6 (Fourier coefficients of a Siegel modular form)

$(Q^2, P^2)\backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	-209304	648	327	0	0	0	0
(2,4)	-2023536	50064	8376	-648	0	0	0
(2,6)	-15493728	1127472	130329	-15600	972	0	0
(4,4)	-16620544	3859456	561576	12800	3272	0	0
(4,6)	-53249700	110910300	18458000	1127472	85176	-6404	0
(6,6)	2857656828	4173501828	920577636	110910300	8533821	153900	26622
(2,10)	-510032208	185738352	16844421	-2023536	315255	-31104	1620

$Q^2, P^2, Q \cdot P$: T-duality invariant bilinears in the charges.

Question: Can we reproduce these numbers from the analysis of quantum gravity?

For this we need to compute quantum gravity / string theory corrections to the Bekenstein-Hawking-Wald formula exactly.

We shall not be able to provide a complete answer to this problem today.

Nevertheless we shall use the black hole description to make predictions for the microscopic index which can be tested against the explicit results.

We shall discuss two examples:

1. Qualitative predictions:

Sign of the index

2. Quantitative predictions:

Logarithmic corrections to the entropy

The key insight arises from the existence of the AdS_2 factor in the near horizon geometry of extremal black holes.

AdS_2 does not admit any charge or energy carrying excitations since such an excitation will change the asymptotic boundary condition on the gauge fields / metric .

$\Rightarrow$ quantum gravity in the near horizon geometry of extremal black holes describes a microcanonical ensemble of degenerate quantum states – the ground states of the black hole in a fixed charge sector.

Sign of the Index

Typically in the microscopic theory we do not calculate the degeneracy, but an index:

$$\Omega \equiv \mathrm{Tr}'(-1)^{F} = \mathrm{Tr}'(-1)^{2J_3}$$

$'$ denotes removal of the trace over fermion zero modes associated with broken supersymmetry.

This is what is protected from corrections when gravity effects are switched on.

For comparison, on the black hole side also we must compute the index, not entropy.

How to compute $\mathrm{Tr}'(-1)^{2J_3}$ for a black hole?

1. SUSY + SL(2,R) isometry of $AdS_2 \Rightarrow SU(1,1|2) \supset SU(2)$

$\Rightarrow$ black holes have spherically symmetric horizon and hence zero average angular momentum.

2. SInce extremal black holes describe a microcanonical ensemble, all states in the ensemble have $J_3 = 0$.

Thus black holes have

$$\mathrm{Tr}'(-1)^{2J_3} = \mathrm{Tr}'(1) = e^{S_{BH}}$$

S_{BH}: black hole entropy (after stringy and quantum corrections)

A.S.; Dabholkar, Murthy, Gomes, A.S.

$$\mathrm{Tr}'(-1)^{2J_3} = \mathrm{Tr}'(1) = e^{S_{BH}}$$

This explains why the index on the microscopic side can be compared with $e^{S_{BH}}$ on the black hole side.

$$\Omega \Leftrightarrow e^{S_{BH}}$$

But this analysis also makes a non-trivial prediction:

$$\Omega \equiv \mathrm{Tr}'(-1)^{2J_3} > 0$$

Microscopic index must be positive

(no *a priori* reason for this on the microscopic side).

$$\Omega \equiv \mathrm{Tr}'(-1)^{2J_3} > 0$$

– a testable prediction for Fourier expansion coefficients of certain Siegel modular forms which appear as the generating function for the spectrum of quarter BPS states in various $N = 4$ supersymmetric string theories.

Note: This prediction requires us to put complete trust on the gravitational description of the black hole.

A spread in J_3 by even 1/2 unit will completely destroy this prediction.

$$(-1)^{2J_3} = -1 \text{ for } J_3 = 1/2 \quad !$$

A caveat

Macroscopic arguments hold for single centered black holes, but the total index receives contribution from single and two centered black hole solutions.

Denef; A.S.; Cheng, Verlinde; Dabholkar, Guica, Murthy, Nampuri

microscopic index = 1-centered index + 2-centered index

2-centered solutions can have negative index spoiling the earlier arguments

1-centered index= microscopic index – 2-centered index

Strategy Compute separately the contribution to the index from two centered black holes, subtract this from the microscopic index and then test the predictions from AdS_2 geometry.

Given a 2-centered configuration we can compute its contribution to the index

Example: Index of (Q, 0) + (0, P) is

$$(-1)^{Q.P+1}\,|Q.P|\,f(Q^2/2)\,f(P^2/2)$$

f(n) defined through

$$\sum_n f(n) e^{2\pi i n\tau} = \eta(\tau)^{-24}$$

Furthermore supergravity analysis tells us what 2-centered configurations exist at any given point in the moduli space. Denef; Denef, Moore

– can use this to compute total 2-centered contribution to the index.

Results for the total index in heterotic on T^6 from microscopic counting

$(Q^2, P^2)\backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	-209304	648	327	0	0	0	0
(2,4)	-2023536	50064	8376	-648	0	0	0
(2,6)	-15493728	1127472	130329	-15600	972	0	0
(4,4)	-16620544	3859456	561576	12800	3272	0	0
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Red entries: Negative index

Blue entries: $\Delta \equiv Q^2P^2 - (Q.P)^2 < 0$ and hence no single centered black holes

Result for the index after subtracting the contribution from two centered black holes

$(Q^2, P^2)\backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	648	648	0	0	0	0	0
(2,4)	50064	50064	0	0	0	0	0
(2,6)	1127472	1127472	25353	0	0	0	0
(4,4)	3859456	3859456	561576	12800	0	0	0
(4,6)	110910300	110910300	18458000	1127472	0	0	0
(6,6)	4173501828	4173501828	920577636	110910300	8533821	153900	0
(2,10)	185738352	185738352	16844421	16491600	0	0	0

No more negative index or $\Delta < 0$ states.

Such tests have been carried out for many other N=4 supersymmetric string theories where the exact dyon spectrum is known.

One can also prove the positivity of the index in the limit of large charges in all the known examples.

But a general proof or counterexample is still missing.

Logarithmic corrections

The exact microscopic results for the index in $\mathcal{N}=4$ and $\mathcal{N}=8$ supersymmetric string theories allow us to compute systematic correction to the index Ω beyond the large charge limit.

Can we reproduce these corrections from the macroscopic side?

Microscopic results in the limit when all components of the charge are taken to be large:

$$\begin{aligned} \ln\Omega &= \pi\sqrt{\Delta} + \mathcal{O}(1) \quad \text{for N=4} \\ &= \pi\sqrt{\Delta} - 2\ln\Delta + \mathcal{O}(1) \quad \text{for N=8} \end{aligned}$$

$$\Delta \equiv Q^2P^2 - (Q.P)^2$$

Note: This is different from the Cardy limit results when only one component becomes large keeping the other components fixed:

$$\begin{aligned} \ln\Omega &= \pi\sqrt{\Delta} - \frac{m+2}{4}\ln\Delta + \mathcal{O}(1) \quad \text{for N=4} \\ &= \pi\sqrt{\Delta} - 2\ln\Delta + \mathcal{O}(1) \quad \text{for N=8} \end{aligned}$$

m: number of matter multiplets

Strategy for computing S_{BH}

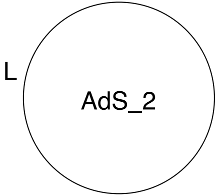
Euclidean near horizon geometry has the form

$AdS_2 \times S^2 \times K$ with flux through various cycles

K: 6-dimensional compact space of string scale size

$$ds^2 = a^2(d\eta^2 + \sinh^2\eta d\theta^2) + a^2(d\psi^2 + \sin^2\psi d\phi^2) + ds_K^2$$

a: a constant that scales with the charges



L: regulated length of the boundary of AdS_2

Let Z_{AdS_2} be the partition function of string theory in this background

Then

$$Z_{AdS_2} = Tr(e^{-L\,H}) = e^{S_{BH} - E_0 L}$$

E_0: energy, S_{BH}: entropy

Once we compute Z_{AdS_2}, we can extract S_{BH} from it.

$$Z_{AdS_2} = e^{S_{BH} - E_0 L}$$

Classical contribution to S_{BH} gives us back the Wald entropy $\pi\sqrt{\Delta}$

One can show that logarithmic corrections to S_{BH}, if present, must come from one loop contribution of massless fields to Z_{AdS_2}

This involves two types of contributions:

1. Determinant of the kinetic operator of massless fields after removing the zero modes

2. Contribution from integration over the zero modes.

1. Determinant of the kinetic operator of massless fields

- **Find the quadratic action of massless fields expanded around the near horizon geometry with fluxes.**

- **Find the eigenvalues of the kinetic operator.**

  Use results by Camporesei, Higuchi

- **Take the product of non-zero eigenvalues.**

Previous analysis for simple systems: Mann, Solodukhin; Fursaev; ..

2. Zero mode contribution

- Identify the asymptotic symmetries responsible for the zero modes.

- Change integration over the zero modes to integration over parameters labelling the (super-)group of asymptotic symmetries.

- The Jacobian for this change of variables gives the zero mode contribution to Z_{AdS_2}.

Given Z_{AdS_2} we can isolate the 'infinite part' $e^{-E_0 L}$ and finite part $e^{S_{BH}}$ easily and compute S_{BH}.

Results for logarithmic term in S_{BH}:

Theory	bosonic determinant	fermionic determinant	zero mode	total
N=4	$\frac{202+41m}{180}\ln\Delta$	$\frac{17+m}{45}\ln\Delta$	$-\frac{1}{4}(6+m)\ln\Delta$	0
N=8	$\frac{224}{45}\ln\Delta$	$\frac{1}{45}\ln\Delta$	$-7\ln\Delta$	$-2\ln\Delta$

m: number of matter multiplets

The final result is in perfect agreement with the microscopic results.

Banerjee, Gupta, A.S.; Banerjee, Gupta, Mandal, A.S.

Future goals:

1. Try to carry out the gravitational path integral exactly using localization techniques.

Banerjee, Banerjee, Gupta, Mandal, A.S.; Dabholkar, Gomes, Murthy

2. Generalize to N=2 supersymmetric string theories

The main bottleneck is the absence of reliable results on the microscopic side.

Nevertheless one can try to make progress on the macroscopic side so that comparison with the microscopic data may be made if and when the latter is available.

Progress has been made on several fronts.

a) A general formula relating the total index to the index associated with single centered black holes has been found.

Manschot, Pioline, A.S.; Kim, Park, Wang, Yi

– required for testing positivity of the index etc.

b) Logarithmic corrections have been computed in the STU model and was found to vanish.

– consistent with the earlier proposal for the index

David; Cardoso, David, de Wit,Mahapatra

c) Computation of logarithmic corrections to half BPS black holes in generic N=2 supersymmetric string theory is in progress.

– would constrain the measure in the OSV integral.

3. Computation of logarithmic corrections can also be extended to non-supersymmetric extremal black hole, *e.g.* extremal Reissner-Nordstrom, extremal Kerr, extremal BTZ etc.

A.S., work in progress

These results would put strong constraint on any microscopic theory that attempts to provide a statistical interpretation of the entropy.

***e.g.* for extremal Kerr we get a correction**

$$\frac{16}{45} \ln A_H$$

Can we reproduce this from Kerr/CFT or some other microscopic theory?

Summary

Quantum gravity in the near horizon geometry can make non-trivial prediction for the microstates which can be tested by explicit microscopic calculation.

1. Positivity of the index

2. Coefficient of the logarithmic correction to the Bekenstein-Hawking-Wald formula in the large charge limit.

There are many other tests not discussed here *e,g.* analysis of twisted index, one loop correction to the entropy from stringy states etc.

Cardoso, de Wit, Kappeli, Mohaupt; David, Jatkar, A.S., ...

General lesson:

Euclidean quantum gravity / string theory is capable of reproducing the microscopic results in all cases which have been analyzed.

Thus while we search for holographic description of gravity in general backgrounds, it may also be fruitful to attempt to study quantum theory of gravity head on, using the known exact results from holography / black hole microstates as useful guiding principle.