Holographic Minimal Models

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Based on

- M. R. Gaberdiel and R. G. , *"An AdS_3 Dual for Minimal Model CFTs,"* arXiv:1011.2986
- M. R. Gaberdiel, R. G., T. Hartman and S. Raju, *"Partition Functions of Holographic Minimal Models,"* arXiv:1106.1897

A Pair of Questions I : Boundary

- The space of large N (not-necessarily-supersymmetric) 2d QFTs is very rich. Perhaps the best understood examples we have of the variety of non-trivial dynamical phenomena in QFT.
- E.g. Sigma models/Principal Chiral models, Gross-Neveu model, 't Hooft model of 2d QCD.
- And, of course, 2d CFTs which are the endpoints of RG flows in this space.

- Can we understand these theories (and their nontrivial features) holographically? Can we extend our AdS/CFT understanding to these examples?
- Are there new features and new lessons to be learnt in non-SUSY cases?

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A Pair of Questions II : Bulk

- Are there complete, consistent theories of (AdS) quantum gravity which do not have a stringy set of additional excitations?

- If the answer is yes, 3d is a good place to look for it - gravity is non-propagating and yet has black holes.
- However, the prospects for pure 3d gravity (supergravity) to be consistent and complete appear dim. Witten [07], Gaberdiel, Maloney-Witten.
- Could a higher spin gravity theory be quantum mechanically well defined?
- Does the dual CFT provide this definition? Or is there an autonomous definition?

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Minimal Model Holography

Proposal: $\mathcal{W}_N$ *minimal model series of 2d CFTs in the large* N*, 't Hooft limit* $\leftrightarrow$ *Vasiliev higher spin theory on* AdS_3 *together with two complex scalars.*

(See Chang-Yin for a modified proposal involving only a modular noninvariant subsector of the $\mathcal{W}_N$ model dual to a theory with one complex scalar.)

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The CFT: A coset WZW theory (generalising the Virasoro unitary series)

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}.$$

A line of fixed points (labelled by $0 \leq \lambda = \frac{N}{N+k} \leq 1$) with $c_N(\lambda) = N(1 - \lambda^2)$ - vector like model.

The Bulk: Fields of spin $s = 2, 3, \ldots \infty$ in AdS_3 coupled to two complex scalars of equal mass.

$$M^2 = -1 + \lambda^2.$$

but quantized oppositely. Correspond to basic primaries $h_{\pm} = \frac{1}{2}(1 \pm \lambda)$.

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Checks I: Symmetries

- Why might something like this be true?
- At least in the large k limit ($\lambda = 0$), these CFTs are essentially those of $(N-1)$ free fermions with a singlet condition.
- This has a large $\mathcal{W}_N$ type higher spin global symmetry. Should be reflected in a large gauge invariance in the dual bulk description.
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- The large N limit of this symmetry is subtle. Asymptotic symmetry algebra of higher spin theories labelled by one parameter: $\mathcal{W}_\infty[\lambda]$. (Gaberdiel-Hartman, Figueroa O'Farill et.al.)
- Exact higher spin symmetry algebra is the wedge algebra $hs[\lambda]$.
- At first sight different from the symmetry of the large N, 't Hooft limit of the $\mathcal{W}_N$ minimal models (with wedge subalgebra $sl(N)$).
- Nevertheless, strong evidence for the equivalence of these two symmetry algebras (generalized level-rank duality Kuniba et.al, Altschuler et.al.).

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The Bulk Spectrum

Can the "infinite N" CFT reproduce the bulk physical spectrum of linearised fluctuations of the higher spin fields?

Perturbative bulk spectrum given by

$$Z_{\rm bulk} = Z_{class} Z_{1-loop} = (q\bar{q})^{-c/24} Z_{\rm HS} Z_{\rm scal}(h_+)^2 Z_{\rm scal}(h_-)^2.$$

where Z_{HS}, Z_{scal} are the bulk one loop determinants from the higher spin fields $(s = 2, 3 \ldots, \infty)$ and scalars resp.

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$$Z_{HS} = \prod_{s=2}^{\infty}\prod_{n=s}^{\infty}\frac{1}{|1-q^n|^2} = \prod_{n=1}^{\infty}|1-q^n|^2 \times \prod_{n=1}^{\infty}\frac{1}{|(1-q^n)^n|^2} \equiv |\tilde{M}(q)|^2.$$

M. R. Gaberdiel, R. G., A. Saha

$$\begin{aligned} Z_{scal}(h) &= \prod_{l=0,l'=0}^{\infty}\frac{1}{(1-q^{h+l}\bar{q}^{h+l'})} \\ &= \exp\left[\sum_{n=1}^{\infty}\frac{Z_{\text{sing par}}(h,q^n,\bar{q}^n)}{n}\right] \\ &= \sum_R \chi_R^{u(\infty)}(z_i)\,\chi_R^{u(\infty)}(\bar{z}_i) \qquad (z_i = q^{i+h-1}). \end{aligned} \tag{1}$$

where $Z_{\text{sing par}}(h,q,\bar{q}) = \frac{q^h\bar{q}^h}{(1-q)(1-\bar{q})}$. (Giombi-Maloney-Yin)

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Putting it all together:

$$Z_{\mathrm{bulk}} = (q\bar{q})^{-c/24}|\tilde{M}(q)|^2 \sum_{R_\pm, S_\pm} |\chi_{R_+}(z_i^+)\,\chi_{S_+}(z_i^+)\,\chi_{R_-}(z_i^-)\,\chi_{S_-}(z_i^-)|^2.$$

$R_\pm, S_\pm$ are representations of $U(\infty)$ with a finite number of boxes in the Young Tableaux. ($z_i^\pm = q^{i+h_\pm-1}$).

View this as the combined contribution from (weakly coupled) multi-particle states of the complex scalar with dimension h_+ (the pieces R_+, S_+), and that of the scalar with dimension h_- (the pieces R_-, S_-) all dressed with the boundary graviton excitations in $\tilde{M}(q)$.

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Spectrum of Primaries

- Primaries in the CFT labelled by two representations (Λ^+, Λ^-) of $SU(N)_k$ and $SU(N)_{k+1}$ respectively.

- $h(\Lambda^+;\Lambda^-) = \frac{1}{2p(p+1)}\left(\Big|(p+1)(\Lambda^+ + \rho) - p(\Lambda^- + \rho)\Big|^2 - \rho^2\right)$ where ρ is the Weyl vector for $SU(N)$

- $h(0;\mathrm{f}) = \frac{(N-1)}{2N}\left(1 - \frac{N+1}{N+k+1}\right) \to \frac{1}{2}(1-\lambda) = h_-;$
  $h(\mathrm{f};0) = \frac{(N-1)}{2N}\left(1 + \frac{N+1}{N+k}\right) \to \frac{1}{2}(1+\lambda) = h_+.$

- $h(0;\mathrm{adj}) = 1 - \frac{N}{N+k+1} \to (1-\lambda)\ ; \qquad h(\mathrm{adj};0) = 1 + \frac{N}{N+k} \to (1+\lambda).$

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Branching Functions

Contribution to $\mathrm{Tr} q^{L_0}$ from each of these primaries (taking into account null states):

$$b_{(\Lambda^+;\Lambda^-)}(q) = \frac{1}{\eta(q)^{N-1}} \sum_{w \in \hat{W}} \epsilon(w) q^{\frac{1}{2p(p+1)}((p+1)w(\Lambda^+ + \rho) - p(\Lambda^- + \rho))^2}$$

$\hat{W}$ is the affine Weyl group (affine translations +usual Weyl reflections)

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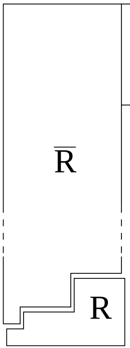
Large N 't Hooft Limit

- To match with bulk spectrum will only consider representations (Λ^+, Λ^-) which are finite tensor powers of fundamentals and/or anti-fundamentals. $\Lambda_\pm = (\overline{R}_\pm, S_\pm)$.



- These representations (i.e. with boxes $B(R), B(S) \sim \mathcal{O}(1)$) have finite dimension in the 't Hooft limit with $k, N \to \infty$.
- Typically, primaries for representations $\Lambda_\pm$ with $\mathcal{O}(N)$ boxes have dimensions that scale as a positive power of N - hence decouple.
- However, when $\Lambda_\pm$ differ by $\mathcal{O}(1)$ boxes, the dimension is finite.
- Though there are many such states (exponential) most of them decouple (in, say, 2,3 point functions) from perturbative states (with $\mathcal{O}(1)$ boxes) - even for large but finite N. Due to fusion rules of CFT.
- Even for primaries with $\mathcal{O}(1)$ boxes there is a large degeneracy.
- E.g. all primaries with $\Lambda_+ = \Lambda_-$ have dimensions $\mathcal{O}(\frac{1}{N})$.
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Reducibility

- The 't Hooft limit needs to be carefully defined to understand the reducibility of representations.
- E.g. for $\Lambda_+ = \Lambda_- = \mathrm{f}$

$$b_{(\mathrm{f};\mathrm{f})} = q^{-\frac{c}{24}}(1 + q^2 + \cdots) + q^{-\frac{c}{24}}(q + 2q^2 + \cdots)\ .$$

  with contributions from vacuum (ω) and adjoint (ψ) primaries.
- However, in the large N limit, there is a natural limit of the operator algebra of these states in which

$$L_1\psi = \omega. \tag{4}$$

- But $\psi \neq L_{-1}\omega$: the representation is reducible but *indecomposable*.
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Checks II: Matching Perturbative Spectra

- Pattern in this reducibility: In $\Lambda_+ \otimes \bar{\Lambda}_-$ the only Λ which do not decouple are the ones where no boxes and antiboxes are annihilated into singlets.
- i.e. $B(\Lambda) = B(\Lambda_+) + B(\Lambda_-)$
- Thus need to correct the CFT partition function in the 't Hooft limit to subtract out these additional null states.
- Since $N^{\Lambda}_{\Lambda_+ \bar{\Lambda}_-}$ become Clebsch-Gordon coefficents, corrected branching function becomes

$$\mathrm{ch}^{\mathrm{cft}}_{R_+ S_+ R_- S_-}(q) = q^{-\frac{c}{24}} \cdot \tilde{M}(q) \cdot \chi_{R_+^T}(z_i^+) \chi_{S_+^T}(z_i^+) \chi_{R_-^T}(z_i^-) \chi_{S_-^T}(z_i^-) \,. \quad (5)$$

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Comparing with the perturbative gravity answer

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$$Z_{CFT}(\lambda) = Z_{bulk}(\lambda)$$

for all values of the 't Hooft coupling λ.

Note that the representations on both sides are related by a transpose.

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- Compare CFT three point function of two scalar primaries and one spin s current $J^{(s)}$ i.e.

$$\langle \mathcal{O}_{\pm} \bar{\mathcal{O}}_{\pm} J^{(s)} \rangle$$

with bulk three point function of two scalars and one spin s gauge field.

- Boundary computation performed for $s = 2, 3$ but for any value of the 't Hooft coupling.
- Bulk computation for any spin s but only for $\lambda = \frac{1}{2}$ ("undeformed theory").
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Where To?

- Understand better the role of the additional light (degenerate) primaries at large but finite N. Can one view them as an almost decoupled sector? Do they contribute at leading order to correlation functions? What is the bulk interpretation? Proposal needs modification? Tests.
- Generalisations I: Bulk duals for cosets involving other Lie groups (see Ahn; Gaberdiel-Vollenweider). More general cosets/ RCFTS (see Kiritsis). Supersymmetric examples.
- Generalisations II: Duals for nonconformal QFTs obtained by RG flow (In Progress).
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