3d theories from 3d manifolds

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to appear

Introduction

Previous talk by prof. Gukov:

The T[M] theories

- 3d N=2 SCFT associated to 3-manifold
- IR limit of A_1 6d (2,0) SCFT on M

Expected properties of T[M]

Space of $R^2 \times S^1$ vacua of T[M]

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SL(2,C) flat connections on M

- Follows from 6d on S^1 == 5d SYM

Expected properties of T[M]

Ellipsoid partition function of T[M]

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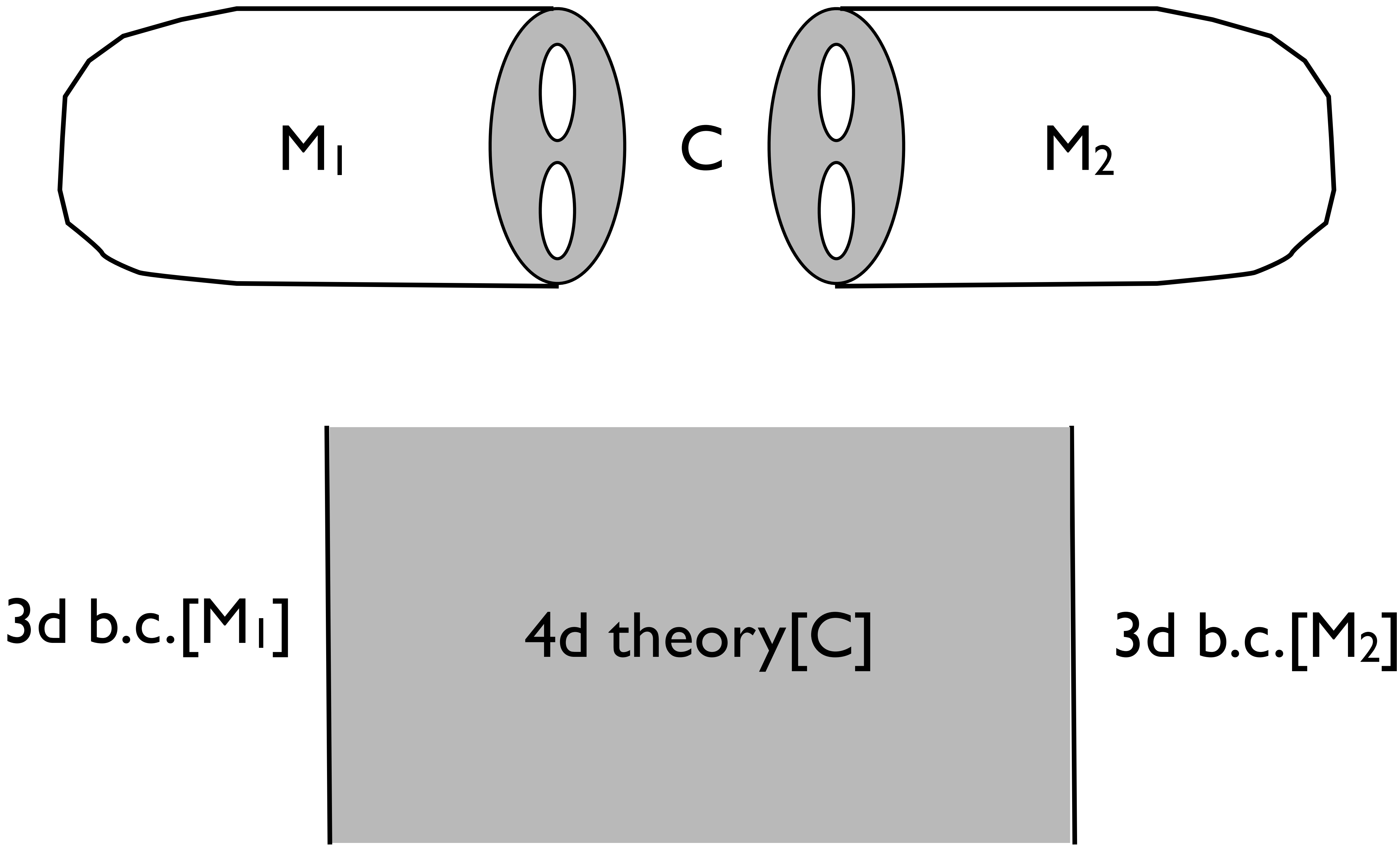
SL(2) CS partition function on M

- Motivated by AGT, Nekrasov-Witten

Motivation

The 6d theory is mysterious

Can we define T[M] directly in 3d?



M1
C
M2
3d b.c.[M1]
4d theory[C]
3d b.c.[M2]

Simple cutting

Cut M along Riemann surfaces?

Too many unknown building blocks

Main result

Explicit three-dimensional construction

of a class of 3d N=2 SCFTs

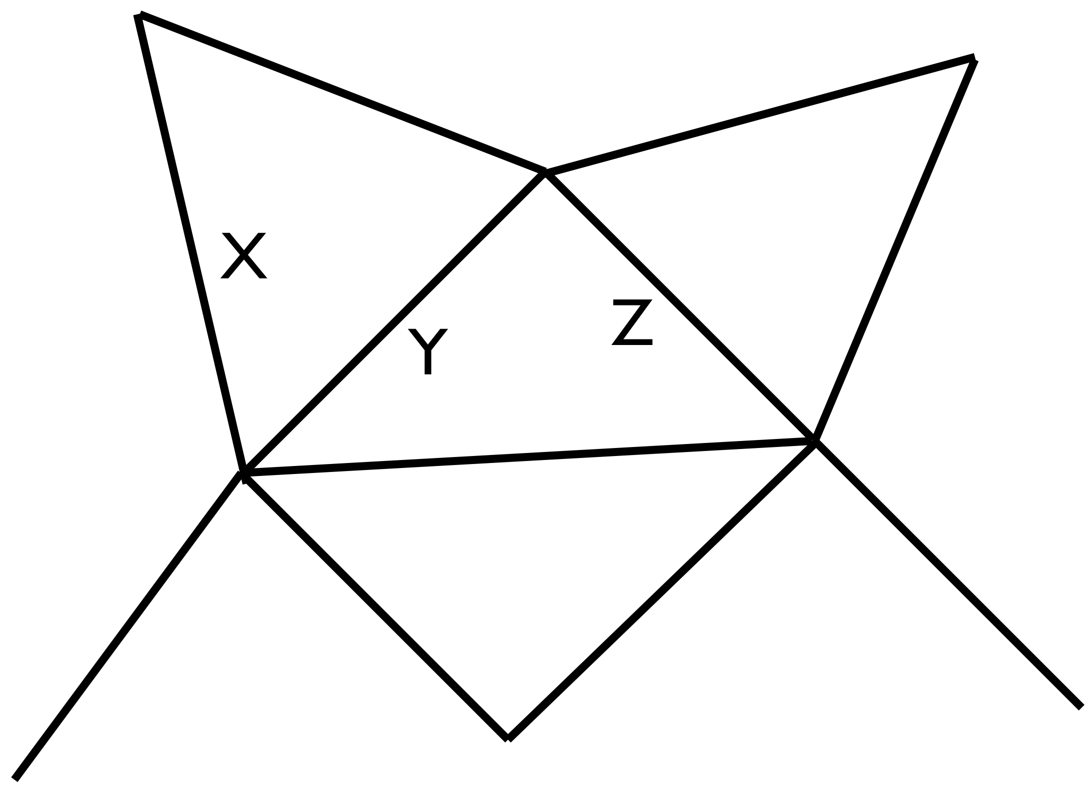
labeled by the same data as

SL(2) Chern-Simons wavefunctions

Labels

- 3d manifold M with boundary + knots
- triangulation of boundary
- ``polarization" of the boundary

Polarization



$[X,Y]=h$

$[Z,Y]=h$

$[X,Z]=0$

....

$$Y\ \Psi(X,Z,\ldots) = -h(\partial_X + \partial_Z)\ \Psi(X,Z,\ldots)$$

....

Main result

SL(2) CS wavefunction on M

equals

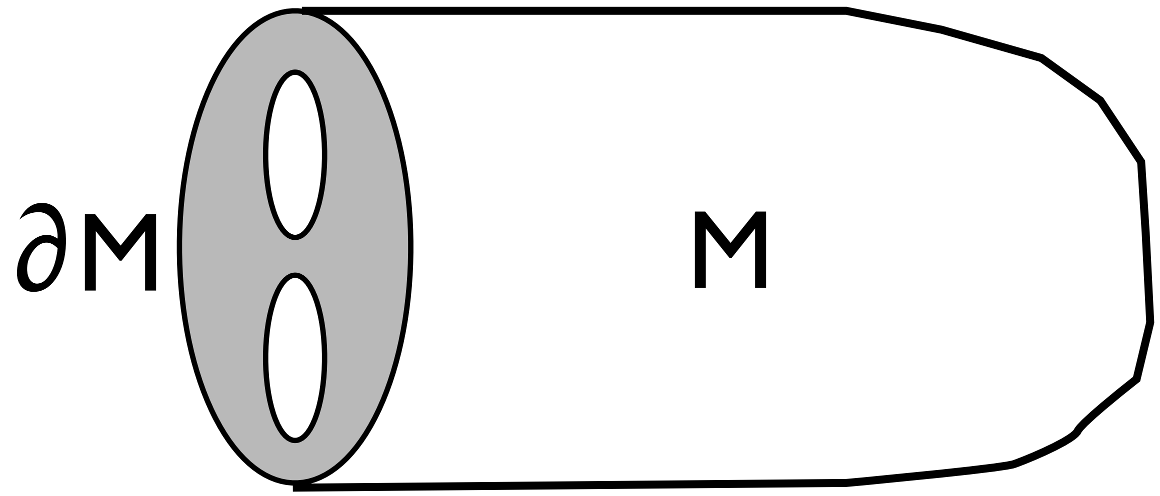
Ellipsoid partition function of 3d SCFT

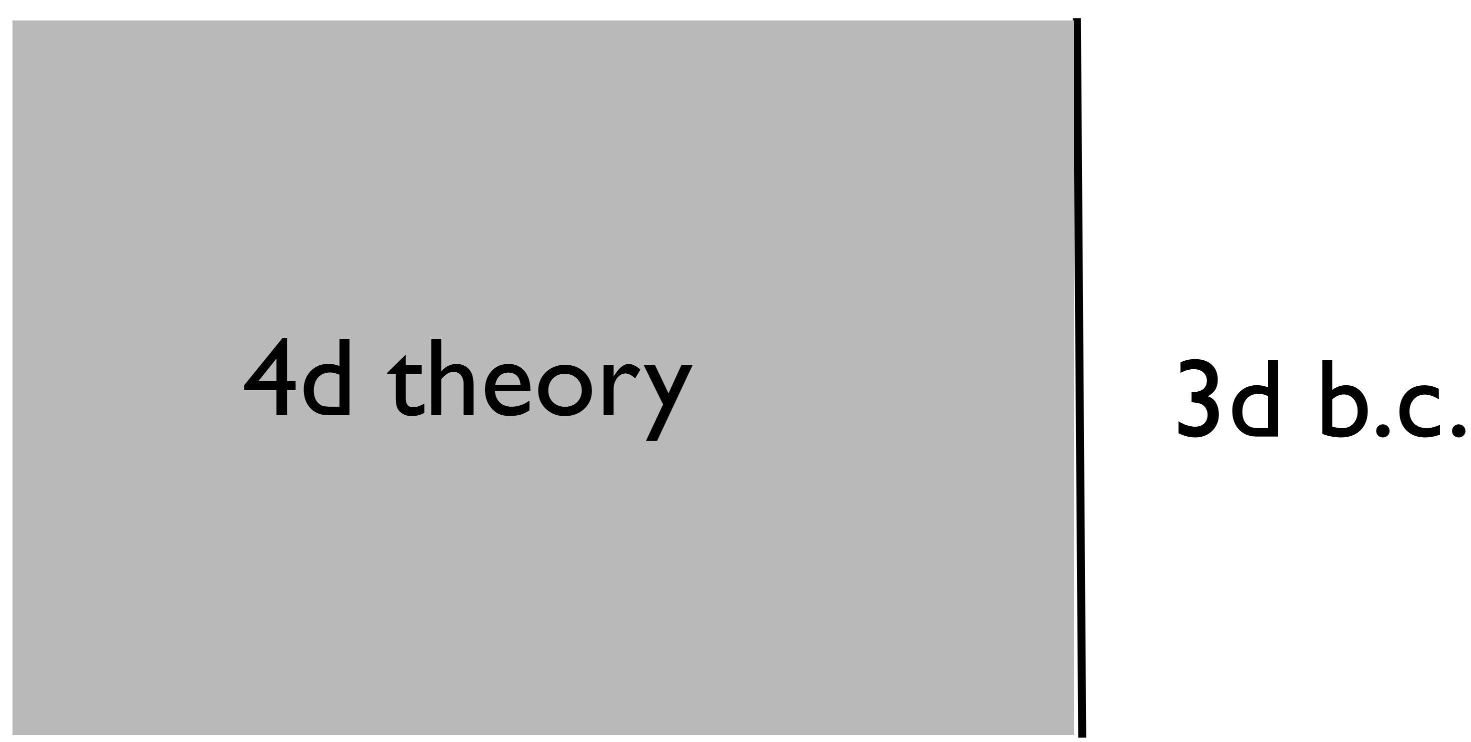
Main conjecture

If M has no boundary, only knots

3d SCFT labeled by M

coincides with T[M]


∂M
M


4d theory
3d b.c.

Main conjecture

If M has boundary ∂M

3d SCFT [M] + 4d SW theory [∂M]

coincides with the

IR limit of A_1 6d (2,0) SCFT on M

Main tool

Decompose M into tetrahedra

- SL(2) CS wavefunction from glued tetrahedra
  Dimofte
- 3d theory from "glued" chiral multiplets.
  - Abelian gauge fields and superpotentials

Consistency conditions

Different decompositions

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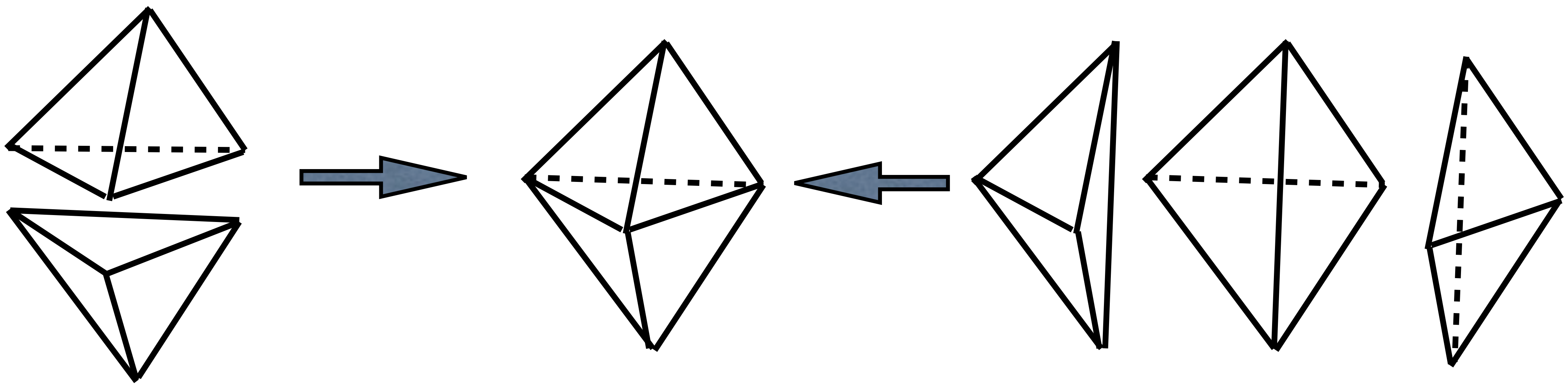
mirror 3d theories

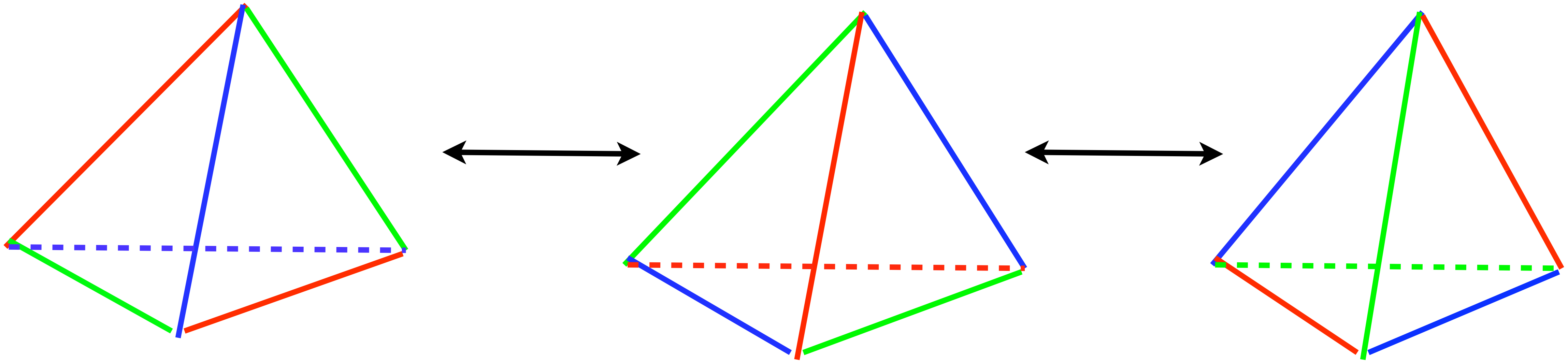
Consistency conditions



$N_f = 1$ SQED <==> 3 chirals; W=XYZ

Aharony, Hanany, Intriligator, Seiberg, Strassler

Consistency conditions



$U(1)_{1/2}$ + 1 chiral == 1 chiral == $U(1)_{-1/2}$ + 1 chiral

Generalizations

Line defects in 3d SCFT

- Labeled by CS Wilson loops in M

Higher rank?

Ellipsoid partition function

- 3d N=2 SUSY gauge theory on ellipsoid
  - $b^2 |z|^2 + b^{-2}|w|^2 = 1$
  - Computable in UV by localization
- Will denote as Ψ_b

Kapustin, Willet, Yaakov
Hama, Hosomichi, Lee

Ellipsoid partition function

- U(1) Flavor symmetry ==> parameter x
  - $x = m + i\,(b+b^{-1})\,R$
    - m: twisted mass
    - R: R-symmetry assignment

Jafferis

  - $\Psi_b(x)$ holomorphic in x

Ψ_b in Abelian 3d theories

Chiral multiplet partition function

$$\Psi_b(x_a)^{\mathrm{chiral}} = s_b(i\,Q/2 - x_a)$$

$Q = b + b^{-1}$

$$s_b(x) \equiv \prod_{m,n \in \mathbb{Z}_{\geq 0}} \frac{mb + nb^{-1} + \frac{Q}{2} - ix}{mb + nb^{-1} + \frac{Q}{2} + ix},$$

Ellipsoid partition function

- Almost unaffected by superpotential W
  - W can break flavor symmetries
  - W must have R-charge 2

Ellipsoid partition function

- Example: W=XYZ
  - x,y,z parameters for $U(1)_X$ $U(1)_Y$ $U(1)_Z$
  - $x+y+z = 0 + i\,(b+b^{-1}) = i\,Q$
  - Adding W constrains $z = iQ-x-y$
- $\Psi_b = \Psi_b(x)^{\text{chiral}}\, \Psi_b(y)^{\text{chiral}}\, \Psi_b(iQ-x-y)^{\text{chiral}}$

Ψ_b in Abelian 3d theories

- Gauge multiplets ===> y_i scalar fields
  - q_a^i charge of chiral multiplet ``a''
  - $x_a = q_a^i y_i + q_a^f z_f$
- Topological currents $*F_i$ give extra flavor
  - FI parameters are twisted masses z'_i

Ψ_b in Abelian 3d theories

- $\Psi_b(z,z') = \int \Psi_b(x_a)^{chiral} \; e^{-i\pi(y,y) - 2 i\pi z'.y} \, dy_i$
- (y,y) Chern-Simons pairing
  - $(y,y) = k\, y^2$ for gauge field level k

$N_f=1$ SQED

$$\int \Psi_b(z+y)^{chiral}\, \Psi_b(z-y)^{chiral}\;\; e^{-i\pi(y,y)\,-\,2\,i\pi\, z'.y}\, dy$$

Things to remember

- Gauging a flavor symmetry
  - Fourier transform with gaussian kernel
  - $Sp(2N,Z)$ on $(2\pi x, -i\partial_x)$
- Adding superpotential
  - linear constraint.

Comparison with wavefunctions

- Dimofte rules
  - tetrahedron => quantum dilogarithm
  - gluing => linear constraints on arguments
  - changes of polarization => Fourier transform

Ψ_b in Abelian 3d theories

tetrahedron => quantum dilogarithm $e_b(i\, Q/2 - x_a)$

- $e_b(x) = s_b(x) \exp i\pi\, x^2/2$
  - Chiral multiplet
  - background CS coupling level -1/2
    - physically required: cancel anomaly

Comparison with wavefunctions

- Pick a 3-manifold decomposed into tetrahedra
- Build 3d theory with $\Psi_b(z)$ = wavefunction
  - tetrahedron ==> chiral multiplet (+CS term)
  - changes of polarization ==> gauging
  - internal edges ==> superpotential terms

Conclusions

- We conjecture a 3d definition of T[M]
  - Many mirror descriptions
  - All are Abelian CSM theories. Why?
- 3d Field theories as 3-manifold invariants!
- We have no direct 6d to 3d derivation

Refined statement

Boundary conditions for N=2

4d Abelian gauge theories

- 3d manifold M with boundary + knots
- triangulation of boundary
- ~~polarization~~

Refined statement

Boundary conditions for N=2

4d SW theories (with BPS particles)

- 3d manifold M with boundary + knots
- ~~triangulation of the boundary~~