Uses of 3d toric varieties

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Strings 2011, Uppsala

SCTV

Based on

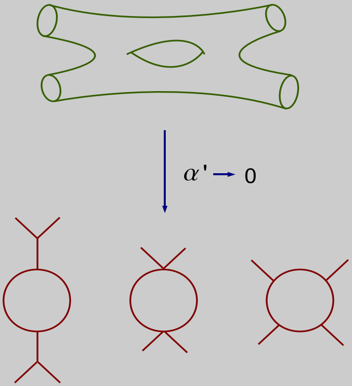
Flux compactification on
smooth, compact, three-dimensional toric varieties

- ♣ M. Larfors, D. Lüst, D. T., JHEP 1007
- ♣ In progress

Outline

- Introduction
- 3d SCTV
- Symplectic quotient
- SU(3) structures
- Conclusions

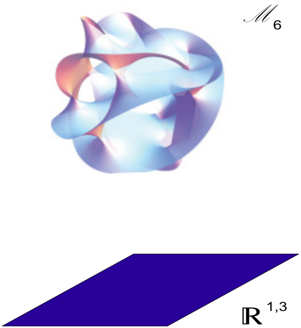
String Theory vs Field Theory



Low-energy limit

- Effective description
- Supergravity solutions

Supergravity solutions

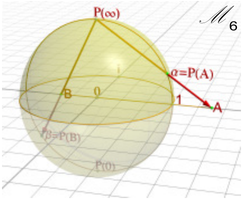


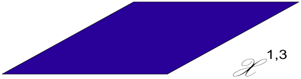
Absence of flux

- Susy vacua $\mathbb{R}^{1,3} \times \mathcal{M}_6$ with $\mathcal{M}_6 = \mathrm{CY}$
- Use math.AG for $\mathcal{M}_6$

♣ Candelas, Horowitz, Strominger, Witten, ’85
♣ Strominger, Witten, ’85
♣ De Wit, Smit, Dass, ’87
♣ Maldacena, Nuñez, ’00

Supergravity solutions





Presence of flux

- Susy vacua $\mathcal{X}^{1,3} \times \mathcal{M}_6$ with $\mathcal{M}_6 \neq$ CY
- Moduli stabilization, susy-breaking, KKLT, ...
- $\mathcal{X}^{1,3} = AdS_4$

♣ Freund, Rubin, ’80

♣ Duff, Pope, ’82

♣ Nilsson, Pope, ’84

♣ Sorokin, Tkatch, Volkov, ’84

Author: J.C. Benoist
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Flux backgrounds

Modern tools

- G-structures and generalized geometry

♣ Gauntlett, Kim, Martelli, Waldram, '01

♣ Gauntlett, Martelli, Pakis, Waldram, '02

♣ Graña, Minasian, Petrini, Tomasiello, '04; '05

Backreaction may be severe

- Susy 'selects' on $\mathcal{M}_6$
  a *non*-integrable almost-complex structure

Main idea

- $\mathcal{M}_6$ may admit another
  integrable almost-complex structure
- Use the underlying
  complex analytic (algebro-geometric) description

3d SCTV

Proposal

- Use directly as internal manifolds in flux compactifications

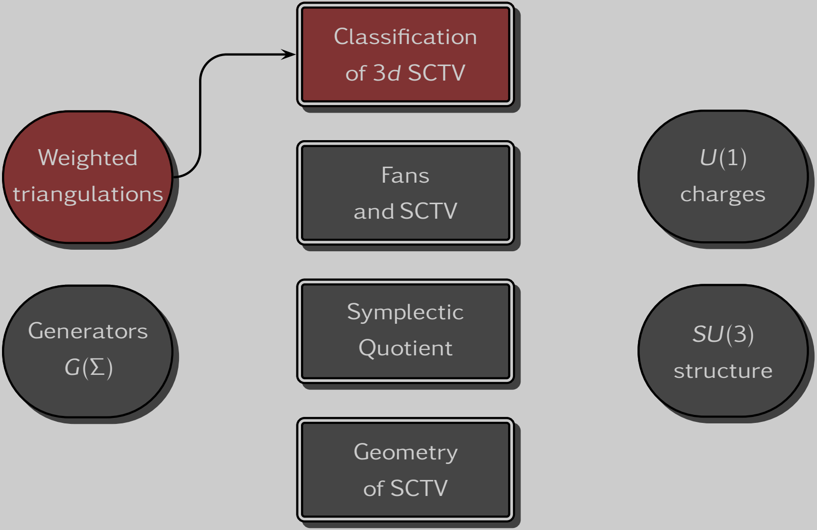
Indirect uses

- No compact toric CY's
- Embedding spaces for CY submanifolds
- Non-compact CY's

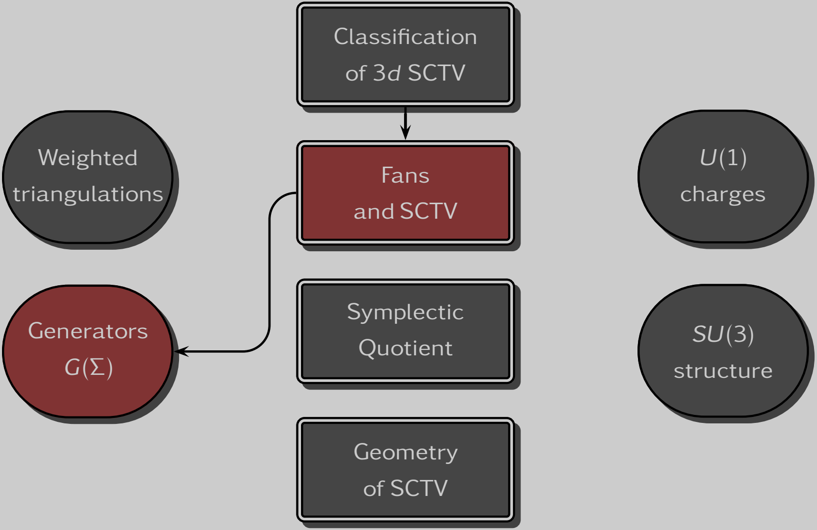
Playground of infinitely many topologies

- Explicit description

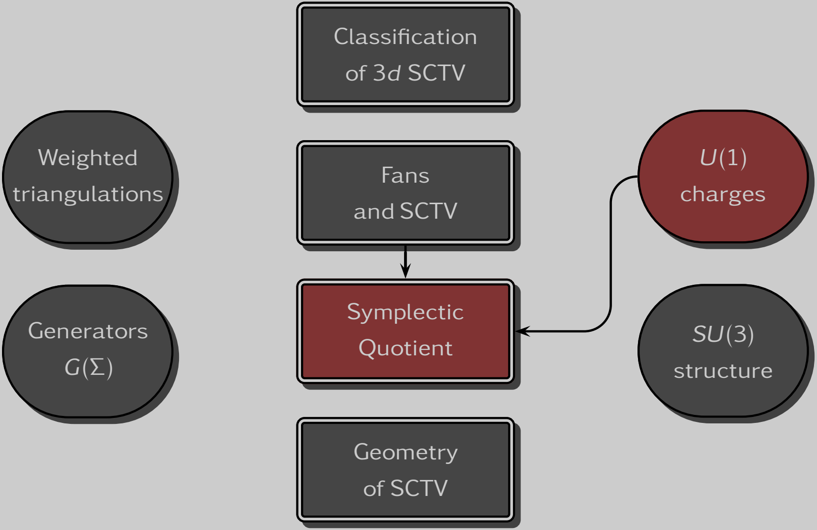
From SCTV to G-structures



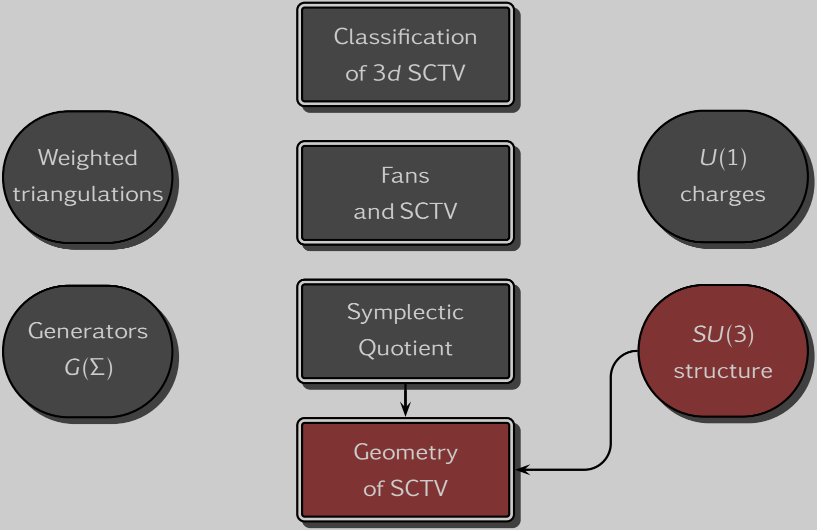
From SCTV to G-structures



From SCTV to G-structures



From SCTV to G-structures



Toric varieties

Fan $\Sigma = \{\sigma_1, \ldots, \sigma_k\} \leftrightarrow V_\Sigma$

Collection of strongly convex cones σ in $N_{\mathbb{R}} := \mathbb{R} \otimes N$, $N \cong \mathbb{Z}^d$

$$\sigma = \{a_1 v_1 + \ldots a_r v_r \; ; \quad 0 \leq a_1, \ldots a_r \in \mathbb{R}\}$$

such that $v_1, \ldots, v_r \in N$ linearly-independent, primitive and

- if $\sigma \in \Sigma$ and $\sigma' \leq \sigma$ then $\sigma' \in \Sigma$;
- if $\sigma, \sigma' \in \Sigma$ then $\sigma \cap \sigma' \leq \sigma$ and $\sigma \cap \sigma' \leq \sigma'$.

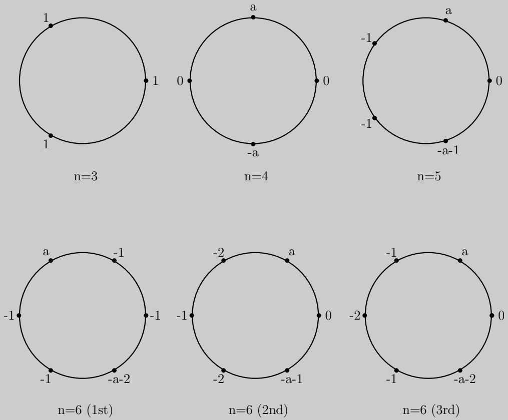
Cone generators $G(\Sigma)$

$$G(\Sigma) = \{v_1, \ldots, v_n\}$$

2d classification

- ♣ Miyake, Oda, reported in Oda, '78

Correspondence: admissible wcg ⟷ 2d SCTV



3d classification

♣ Miyake, Oda, reported in Oda, ’78

d=3, $N \cong \mathbb{Z}^3$

$S^2 \subset N$, centered at the origin

Canonical isomorphisms between

- 3d SCTV
- admissible double $\mathbb{Z}$-weightings of S^2
- admissible N-weightings of S^2

3d classification

N-weighting

- Triangulation of S^2 by spherical triangles
- Assignment of primitive $v \in N$ to each spherical vertex

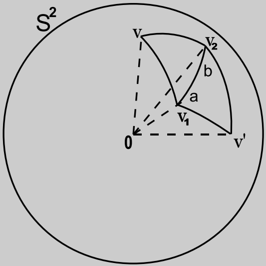
Admissible N-weighting

- Intersect a fan Σ with the sphere S^2 $\Rightarrow$ triangulation
- Vertex of the triangulation $\leftrightarrow$ generator in $G(\Sigma)$

3d classification

Double $\mathbb{Z}$-weighting

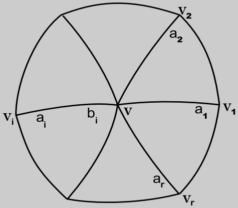
Assignment of a pair of integers to each spherical edge



Admissible N-weighting $\Rightarrow$ double $\mathbb{Z}$-weighting

$$v + v' + a v_1 + b v_2 = 0$$

3d classification



Admissible double $\mathbb{Z}$-weighting

- The equations

$$v_{i-1} + v_{i-1} + a_i v_i + b_i v = 0$$

  are compatible for each v
- The weighted link of each v is an admissible wcg
- Can solve to determine $G(\Sigma)$

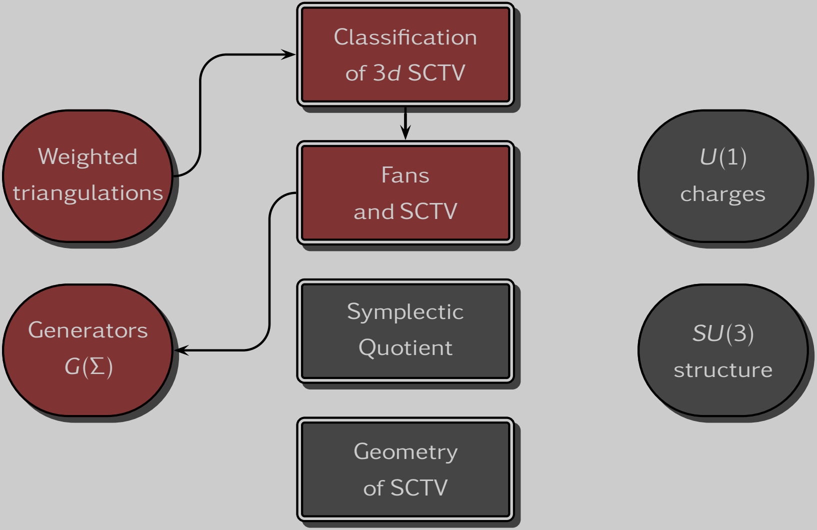
3d classification

♣ Miyake, Oda, reported in Oda, '78

Partial classification of (minimal) 3d SCTV

- $\mathbb{CP}^2$ bundles over $\mathbb{CP}^1$
- $\mathbb{CP}^1$ bundles over 2d SCTV
- Complete N-weightings for triangulations $n \leq 8$

From SCTV to G-structures



Symplectic quotient

Moment maps

$$\mu^a := \sum_{i=1}^{n} Q_i^a |z^i|^2 - \xi^a$$

$$a = 1, \dots s \; ; \quad z^1, \dots z^n \in \mathbb{C}^n \; ; \quad d = n - s$$

$U(1)^s$ action on $\mathbb{C}^n$

$$z^i \longrightarrow e^{i\varphi_a Q_i^a} z^i$$

Toric variety $\mathcal{M}_{2d} = V_\Sigma$

$$\mathcal{M}_{2d} = \mu^{-1}(0)/U(1)^s$$

- Unique topology for $\xi^a \in \mathcal{K}_\mathcal{M}$

Symplectic quotient

Forms on $\mathcal{M}_{2d}$

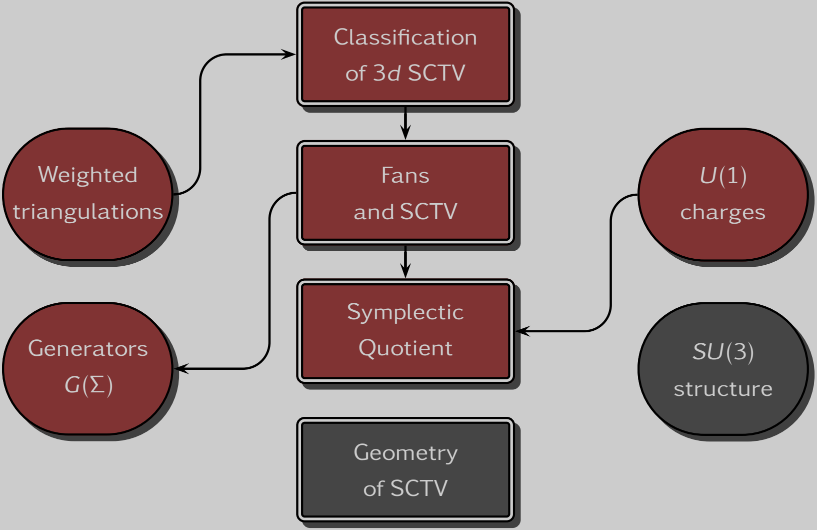
- *Basic* forms on $\mu^{-1}(0)$
- Gauge-invariant forms Φ on $\mathbb{C}^n$ subject to $\mu^a = 0$, $P(\Phi) = \Phi$

Relation to the previous description

- Generators $G(\Sigma) \leftrightarrow U(1)$ charges

$$\sum_{i=1}^{n} Q_i^a v_i = 0\,, \quad a = 1,\ldots,s$$

From SCTV to G-structures



SU(3) structures

Compactifications on $\mathcal{M}_6$

- Susy ‘selects’ an SU(3) or SU(3)$\times$SU(3) structure on open sets
- Global solution by extension
- Convenient to have a global SU(3)

Topological obstruction

- $\mathcal{M}_6$ must be spin

SU(3) structures

$SU(3)$ structure on $\mathcal{M}_6$

- Ω complex decomposable three-form
- J real two-form
- $\Omega \wedge J = 0 \quad \& \quad \Omega \wedge \Omega^* = -\frac{4i}{3} J \wedge J \wedge J \neq 0$

Link with supergravity

- $\epsilon \sim \zeta \otimes \eta$
- $\Omega \sim \eta \gamma_{(3)} \eta \quad \& \quad J \sim \eta^\dagger \gamma_{(2)} \eta$

SU(3) structures

Torsion classes $\mathcal{W}_1, \ldots \mathcal{W}_5$

$$dJ = \frac{3}{2}\mathrm{Im}(\mathcal{W}_1 \Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$
$$d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega$$

Solutions

- Conditions for vacuum $\leftrightarrow$ Conditions on $\mathcal{W}_i$
- Geometrical problem

Necessary and sufficient conditions on $\mathcal{W}_i$

IIA/B $\mathcal{N} = 2$ CY

- $F = 0, \quad \mathcal{W}_i = 0$
- $ds^2 = ds^2(\mathbb{R}^{1,4}) + ds^2(\mathcal{M}_6)$
- Many examples

IIA $\mathcal{N} = 1$ rigid $SU(3)$

- $F \neq 0, \quad \mathcal{W}_3, \mathcal{W}_4, \mathcal{W}_5 = 0, \quad \mathcal{W}_1, \mathcal{W}_2 \neq 0, \quad d\mathcal{W}_2 \propto \mathrm{Re}\Omega$
- $ds^2 = ds^2(AdS_4) + ds^2(\mathcal{M}_6)$

♣ Lüst, D. T., '04

- A handful of examples

♣ Behrndt, Cvetic, '04

♣ Tomasiello, '07

♣ Koerber, Lüst, D. T., '08

SU(3) structures on SCTV

'SCTV' now refers to the topology

Sufficient conditions

$(1,0)$ form K on $\mathbb{C}^n$ such that

- $P(K) = K$
- $Q^a(K) = \frac{1}{2} Q^a(\Omega_{\mathbb{C}})$
- $|K|^2 = 2, \text{ on } \mu^{-1}(0)$

Local $SU(2)$ structure

$$\omega = -\frac{i}{2} K^* \cdot \widetilde{\Omega} \quad \& \quad j = \widetilde{J} - \frac{i}{2} K \wedge K^*$$

where

$$\widetilde{\Omega} \propto \prod_{a=1}^{s} \iota_{V^a} \Omega_{\mathbb{C}} \ \ \& \ \ \widetilde{J} = P(J_{\mathbb{C}}) \, , \quad V^a = \sum_i Q_i^a z^i \partial_{z^i}$$

SU(3) structures on SCTV

Global $SU(3)$ structure

$$J = \alpha j - \frac{i\beta^2}{2} K \wedge K^* \quad \& \quad \Omega = \alpha\beta e^{i\gamma} K^* \wedge \omega$$

Torsion classes

- Generally $\mathcal{W}_i \neq 0$
- Special points where $\mathcal{W}_1, \mathcal{W}_3, \mathcal{W}_4 = 0$

♣ Tomasiello, '07

♣ Gaiotto, Tomasiello, '09

♣ Larfors, Lüst, D.T., '10

SU(3) structures on SCTV

Remarks

- Conditions are easy to satisfy
- A plethora of SU(3) structures on various 3d SCTV
- Does not produce solutions automatically:
  torsion classes must be computed case by case

♣ In progress

SU(3) structures on SCTV

Known 3d SCTV solutions (topology)

Susy $AdS_4 \times \mathcal{M}_6$ vacua, where $\mathcal{M}_6$ is:

- $\mathbb{CP}^3$

♣ Nilsson, Pope, '84

♣ Sorokin, Tkatch, Volkov, '84

♣ Tomasiello, '07

♣ Koerber, Lüst, D. T., '08

♣ Aharony, Jafferis, Tomasiello, Zaffaroni, '10

- S^1 reduction of $Y^{p,q}(\mathcal{B}_4)$
  where $\mathcal{B}_4$=Kähler-Einstein

♣ Gauntlett, Martelli, Sparks, Waldram, '04

♣ Martelli, Sparks, '08

SU(3) structures on SCTV

Known 3d SCTV solutions (topology) cont'd

Susy $AdS_4 \times \mathcal{M}_6$ vacua, where $\mathcal{M}_6$ is:

- Massive deformation of S^1 reduction of $Y^{2,3}(\mathbb{CP}^2) = M^{1,1,1}$

♣ Petrini, Zaffaroni, '09

- Massive deformation of S^1 reduction of $Y^{p,q}(\mathcal{B}_4)$
  where $\mathcal{B}_4$=Kähler-Einstein

♣ Lüst, D. T., '09

- Massive deformation of S^1 reduction of $A^{p,q,r}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

♣ Gauntlett, Martelli, Sparks, Waldram, '04

♣ Chen, Lu, Pope, Vazquez-Poritz, '04

♣ Tomasiello, Zaffaroni, '10

Non-compact example

- Resolved conifold

♣ Chen, Dasgupta, Franche, Katz, Tatar, '10

Conclusions

Summary

SCTV's as compactification manifolds may offer many concrete examples where ideas about flux compactifications and AdS/CFT can be tested explicitly.

- Method to produce SU(3) structures on 3d SCTV
- Does not automatically produce solutions:
  torsion classes must be computed

Future directions

- Searches for:
  AdS_4/dS_4 *vacua, SB vacua*
  KK reductions, low-energy effective actions, ...
- Consistent truncations on 3d SCTV
- Existence theorems for
  specific types of $SU(3)$ structures on 3d SCTV

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(Thank You)